# Accounting of Uncertainties in the Radiative-Equilibrium Approach

**Dan Hughes**
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## Introduction

The radiative-equilibrium approach for estimating the Earth's surface temperature is encountered all over the place. Using interval arithmetic, the effects of uncertainties in the numerical values of the parameters that enter this approach are evaluated.

It is found that for even very small uncertainties in the parameters the range of temperature calculated with the equation bound all those that have been stated to be due to the change in the concentration of CO2 in the atmosphere.

An appendix summarizes a lot of what is wrong with the radiative-equilibrium approach.

## A Radiative-Equilibrium Equation

An equation for the radiative-equilibrium energy exchange for the Earth is almost always stated to be

$$\frac{1}{4}(1-\rho) f_{es} \varepsilon_{Sun} \sigma T_{sun}^4 = (1-\gamma)\sigma T_e^4 \tag{1.1}$$

where

$\rho$ is the reflectivity of the ultra violet by the Earth atmosphere

$\gamma$ is the reflectance of the infra red by the Earth surface

$f_{es}$ is the Earth-Sun view factor

$T_{sun}$ is the effective radiative temperature of the Sun surface

$\sigma$ is the Stefan-Boltzmann constant

$T_e$ is the effective temperature of the Earth surface

$\varepsilon_{Sun}$ is the emissivity for the Sun surface

Some nominal values for the parameters are as follows

$\rho = 0.30$

$\gamma = 0.40$

$T_{sun} = 5776.0$

$f_{es} = 2.1646\text{E-05}$

$L = 3.84194\text{E+26}$

Actually, there should be an emissivity associated with the Earth surface, but the numbers of parameters is large enough already.

The Earth-Sun view factor is

$$f_{es} = \left( \frac{R_{Sun}}{R_{ES}} \right)^2 \tag{1.2}$$

where $R_{Sun}$ is the radius of the Sun, and $R_{ES}$ is the Earth-Sun distance. Nominal values for these are

$$R_{Sun} = 6.960 \text{x} 10^8 \text{ m}$$
$$R_{ES} = 1 \text{AU} = 1.496 \text{x} 10^{11} \text{ m}$$

The luminosity for the Sun surface taken to be a perfect radiator is

$$L_{Sun} = 4\pi R_{Sun}^2 I_{Sun} = 4\pi R_{Sun}^2 \varepsilon_{Sun} \sigma T_{Sun}^4 \tag{1.3}$$

and the power hitting a unit perpendicular area the Earth surface is

$$I_{TOA} = \varepsilon_{Sun} \sigma T_{Sun}^4 \left( \frac{R_{Sun}}{R_{ES}} \right)^2 \tag{1.4}$$

Putting it all together, the temperature of the surface that represents the Earth is

$$T_e = \left[ \frac{(1-\rho)\varepsilon_{Sun}}{4(1-\gamma)} \right]^{1/4} \left( \frac{R_{Sun}}{R_{ES}} \right)^{1/2} T_{Sun} \tag{1.5}$$

In a previous post I took the Earth-Sun view factor to be constant. And I used somewhat-large ranges for the uncertainties. In the present post, I've changed both of these.

## Results

The ranges for the uncertainties for the parameters are as follows. The nominal values for the Sun temperature was taken to be 5776.0 K and varied by 5 K on either side. The Sun radius and Sun-Earth distance were assumed to be known to within 0.05 %. The nominal value for the Sun emissivity was taken to be 0.99 and varied from 0.98 to 1.00. The nominal value for the Earth ultra-violet reflectivity was taken to be 0.30 and varied from 0.2850 to 0.3150. The corresponding numbers for the infra-red reflectance were 0.380 to 0.420. Considering the fuzziness associated with these physical quantities, I consider the range to under-represent the possible ranges.

These values resulting in very small changes in the Earth-Sun view factor, 0.20 %, and luminosity of the Sun, 0.45 %. The energy input at the TOA varied by about 1.56 %. At this point, the important changes begin to show up. The energy striking the Earth surface varied by about 5.84 % and the temperature at the Earth surface by about 2.29 %.

The results indicate that both the reflectance have the dominant influence on the calculated surface temperature. An additional calculation with a 1.0 K variation for the Sun temperature, and all other variations unchanged, showed that the total variation of the Earth temperature was about 9.76 K. The Earth temperature is not sensitive to the Sun temperature.

As is the case with the physical phenomena and processes of other aspects of the climate, those associated with the controlling parameters are among the most difficult to handle from the basic fundamental level. The liquid, vapor, and solid phases of water and the distributions of these vertically, plus other aerosols suspended in the atmosphere, and the interactions of all these with the radiative-energy transport make for an extremely difficult problem. One that is very unlikely to be handled at the fundamental level for the foreseeable future.

The numerical values are summarized in the nearby table.

| | Low | Nominal | high | | Range |
|---:|:---:|:---:|:---:|---|:---:|
| T Sun = | 5771.00 | 5776.00 | 5781.00 | | 10.0000 |
| | | | | | |
| R Sun = | 6.9565E+08 | 6.9600E+08 | 6.9635E+08 | | |
| R ES = | 1.4952E+11 | 1.4960E+11 | 1.4967E+11 | | |
| VF ES = | 2.16023E-05 | 2.16455E-05 | 2.16888E-05 | | |
| | | | | | |
| ε Sun = | 0.980 | 0.990 | 1.000 | | |
| Luminosity = | 3.8249E+26 | 3.8421E+26 | 3.8592E+26 | | |
| | | | | | |
| ρ = | 0.2850 | 0.3000 | 0.3150 | | |
| I TOA = | 1331.53 | 1352.58 | 1373.63 | | 42.10 |
| | | | | | |
| γ = | 0.380 | 0.400 | 0.420 | | |
| At Earth = | 228.02 | 236.78 | 245.54 | | 17.51 |
| T Earth = | 283.79 | 288.83 | 293.95 | | 10.16 |
| | | | | | |
| 1.0K T Sun | | | | | |
| T Earth = | 283.98 | 288.82 | 293.74 | | 9.76 |

Note that the bounding values of the calculated quantities can also be obtained by stacking the uncertainties in the proper way. A sort-of Verification of my coding.

## Conclusions

The uncertainties associated with the parameters in the basic radiative-equilibrium approach overwhelm the expected consequences estimated to obtain by the increasing concentration of CO2 in the Earth atmosphere.

# Appendix A: A Rant

## What’s Wrong with the Radiative-Equilibrium Approach

The radiative-equilibrium concept, the basis of so-called greenhouse warming, is an over-simplified approach to the problem. In the strict sense that the word 'equilibrium' is used in thermal energy balance and transport problems, it means that all materials that make up the system of interest are all at a uniform and constant temperature. That is, the potential for energy transfer is everywhere zero. If the systems of the Earth’s climate were at equilibrium, nothing would ever change; weather wouldn’t happen, for example. The Earth’s systems are not now, have never been, and will never be in equilibrium in this sense. The Earth’s climate systems are always changing on all time scales. The energy transport into and out of and within the systems and between the systems is always changing.

In the greenhouse-warming sense, equilibrium is used to mean that the incoming and outgoing radiative energy transport are equal. This is clearly an hypotheses / assumption in that the temperature at all locations on the Earth's surface is always changing at all time scales.

Consider the usual application of the radiative-equilibrium energy balance equation in which the incoming energy from the Sun is balanced by radiation from the surface of some material that is taken to represent the Earth's surface. The zeroth-order approximation is to take the Earth's surface to be a black body suspended in an atmosphere that is transparent to both the incoming and outgoing radiative-energy transport.

At radiative equilibrium, the power incident on the Earth’s surface must equal the power emitted by the surface. For this situation, the equations given in the text reduce to

$$\frac{1}{4}\left(\frac{R_S}{Au}\right)^2 T_S^4 = T_{erth}^4 \qquad (1.6)$$

For a Sun temperature of 5775.0 K, the solar constant is about 1365.2 W/m^2, and Eq. (1.6) gives the temperature of the radiating surface of the Earth to be about 278.5 K ( about 6 C, 42 F ). This temperature is lower than the approximate observed value of about 288 K ( about 14.5 C, 58 F ).

The Earth’s surface does not behave as a black body. The Earth's surface is almost always taken to be represented by a blackbody surface; it is not and an emissivity should be introduced. If we introduce the emissivity into the right-hand side of Eq. (1.6), and take its value to be 0.87 the temperature of the radiating surface of the Earth is about 288.4 K ( 15.3 C, 59.5 F ).

The next approximation is usually taken to be that some fraction of the incoming radiation is reflected from the atmosphere surrounding the planet and the remainder is absorbed by the planet. For this case, the equations in the text reduce to

$$(1-\rho)\frac{1}{4}\left(\frac{R_S}{R_{ES}}\right)^2 T_{Sun}^4 = T_{earth}^4 \tag{1.7}$$

The usual value for the reflected portion is about 30% with the remaining 70% absorbed by the planet. Equation (1.7) gives the surface temperature to be about 255 K ( about -18 C, -1 F ). The numerical value for the albedo for the entire planet is some kind of time-and-space average because the albedo for the Earth's surfaces and atmosphere are constantly changing. The specific details of the averaging are seldom given when the GHG introductory material is the subject. Ultimately, the numerical values for the albedo can only be obtained from measured, or EWAG'd, information.

The time scales for radiative equilibrium are measured in the hundreds to thousands of years. The albedo at the present time may or may not have any relationship to the albedo at those times.

It is at this point that the standard introduction of the radiative-equilibrium concept has gone off the rails. Introduction of the reflectivity, or albedo, should address the issues about the physical phenomena and processes that are responsible for the reflectivity. The answer, of course, is clouds plus a small amount of reflection from the material at the surface of the Earth. Clouds are the dominate factor that determine the numerical value of the reflectivity, although a host of other solid and liquid particulate matter are present in the atmosphere and have significant effects on the radiative-energy-transport properties of the atmosphere. So much so, that the fundamental radiative-transport aspects of the particulate matter are used to adjust, or tune, model predictions in hindcasts of the thermal response of the Earth's systems.

The reflectivity is not due to the minor so-called Greenhouse Gases. These gases are completely transparent to the incoming UV energy. Photons whiz right these molecules un-bothered by the smaller stuff that make up the molecule.

While the radiative energy transport properties of homogeneous mixtures of pure gases is well understood, the Earth's atmosphere is not such a mixture. The Earth's atmosphere is a heterogeneous mixture of gases, liquid, vapor, and solids.

Instead of introducing the various phases of water, and the all-important clouds and the vertical distributions of the clouds, the introductory material almost always jumps right to the minor greenhouse gases. This is wrong, in my opinion.

Well, while the reflectivity and greenhouse gases have been introduced, the radiative-energy transport aspects of the presence of the interactive atmosphere are almost never mentioned. That is, the thermal effects of the greenhouse gases and interaction with the infra red radiation from the Earth's surface are almost never mentioned.

When the infra red radiation effects are accounted for, as shown in the text, the calculated surface temperature increases to about 288 K. I consider the material presented in the text to be a more nearly complete introduction to the radiative-equilibrium approach. However, I have yet to see this well-known and fundamental issue mentioned whenever the simple radiative-equilibrium approach is introduced.

## Summary

While I understand fully the utility of simple approximations to complex subjects, the really good ones capture only the important aspects of the complex situation. I’ve developed and used many such approximations. The radiative-equilibrium 'model' is seriously flawed right off the starting line because an equilibrium state has never been, and will never be, attained by the Earth's systems. I read 'equilibrium' to have the usual standard meaning of static equilibrium. And I can even accept some kind of quasi-static equilibrium in which the systems are not in strict static equilibrium, but can approach or approximate such a state. For the Earth’s systems, however, the long-range time scale is enormous and almost any time period of interest can be approximated as attaining quasi-equilibrium characteristics.

The radiative-equilibrium balance is an hypothesis, pure and simple. Additionally, the pre-industrial period of Earth's history is taken to be the most recent period of time for which radiative-equilibrium is assumed to be true. So far as I know, it has not yet been shown that this assumption is correct.

The Earth's systems are not now, have never been, and will never be, in thermal equilibrium. The radiative-equilibrium balance that is used to introduce the 'global warming problem' is dominated by the presence of water vapor, liquid water, and solid water in the atmosphere. The default albedo value used in this equation is dominated by the effects of clouds. From time to time, other particulate matter contribute to significant decreases in the amount of energy reaching the Earth's surface.

The energy balance for the planet is not yet known in sufficient detail to determine that an imbalance of a few (less than 5) Watts per square meter does in fact exist. The energy equation models in the GCMs very likely are not sufficiently detailed to produce results that are in accord with the physical world.

We do not yet know if the energy content of the Earth's systems is increasing or decreasing. The data seem to indicate that the energy content of some subsystems has both increased and decreased over the course of several years. The so-called Global Average Surface Temperature ( GAST ) has little to no information about the energy content of the Earth's systems. Changes in the GAST primarily reflects the re-distribution of the energy content of some parts of some of the Earth's systems. Changes in the GAST of some parts of some subsystems cannot indicate changes in the energy content of the complete system. For thermodynamically heterogeneous systems, it is only for states near to equilibrium states that changes in the temperature might correctly reflect changes in the energy content.

The earth's systems are constantly undergoing changes in state on a wide range of time scales; hourly, daily, seasonally, yearly, and various multi-year ( decades, centuries and beyond ) responses. The redistribution of the energy content, on very large scales in time and space, affect the radiative-balance properties of the system.

## Radiative Energy Transport

One of the biggest problems that I have with the radiative-equilibrium statement of the problem is as follows. That is the introduction of the albedo while at the same time introducing CO2 as a greenhouse gas. The albedo is first of all dominated by the various phases of water ( liquid, vapor, and solid ) that are present in the atmosphere. Clouds are sometimes cited when the albedo is introduced, but not always. It is the phases of water that is primarily responsible for the difference between the model-calculated greenhouse-free atmosphere temperature and the observed temperature of Earth. The fact that water vapor is important also for absorption and reflection of radiative energy is almost never mentioned at the level of introduction of the radiative-equilibrium balance stage.

Additionally, there is present in the atmosphere other stuff equally important as, and maybe even more important than, CO2. And that stuff is the things that make the radiative-transport problem for the real Earth an interactive-media radiative-energy-transport problem; one of the more difficult problems relative to the important physical phenomena and processes. The solid and liquid particulate matter act to reflect, emit, and absorb radiative energy. Description of raqdiative-energy transport in even simple homogeneous interacting media is a very difficult problem and can be approached from first principles in only the most idealized cases.

The physical phenomena and processes associated radiative-energy transport and particulate matter makes the radiative-energy transport problem intractable from first principles. The GCMs use the all-encompasing parameterizations to deal with these. And the parameterizations for the phenomena and processes are very likely among the WAG-est of EWAGs. And yet the interaction between radiative transport and particulate matter are also among the dominate controlling parameters for calculating the energy budget for the Earth's systems. Particulate matter parameterizations are almost always introduced whenever the GCMs are tuned so as to more-or-less describe the thermal history of the GAST. Note that the GAST is very likely not the proper

This is another area for which I find the simple radiative-equilibrium model approach to be seriously flawed. It has been known for over 100 years that the phases of water and solid particulate matter are among the more important aspects of the Earth's energy budget. And yet, these have been, for the most part, neglected while CO2 effects have been placed front and center, almost to the exclusion of all other important aspects.